

Chapter 9 / **Example 13****Diagonalisation and powers of a matrix**

- a** Find the diagonalization of $A = \begin{pmatrix} 1 & 2 \\ 3 & 0 \end{pmatrix}$
b Hence find an expression for A^4 in the form PD^4P^{-1} .
c Find an expression for A^4 as a product of 3 matrices with no exponents.

$$|A - \lambda I| = 0 \Leftrightarrow \begin{vmatrix} 1-\lambda & 2 \\ 3 & -\lambda \end{vmatrix} = 0 \Leftrightarrow \lambda^2 - \lambda - 6 = 0$$

Press **MENU** **A** $\frac{\text{EQUA}}{\text{X}^2+\text{X}+\text{C}}$ and press **F2** POLY.

Select **F1** degree 2.

Enter the coefficients of the quadratic equation.

Press **F1** SOLVE.

The calculator displays the solution $\lambda_1 = 3, \lambda_2 = -2$

The eigenvectors are

$$x_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \text{ and } x_2 = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$$

$$P = \begin{pmatrix} 1 & 2 \\ 1 & -3 \end{pmatrix}$$

Press **MENU** **1** $\frac{\text{RUN-MAT}}{\text{X}^2+\text{X}+\text{C}}$ to display the Run-Matrix screen for arithmetical calculations.

Press **F3** \blacktriangleright MAT/VCT **F3** DIM

Change the dimensions of the matrix to 2×2 and press **EXE**.

Enter the values of the elements of the matrix P , using **EXE** to move through the matrix.

Press **EXIT** twice to return to the calculation screen.

Press **OPTN** **F2** MAT/VCT **F1** Mat

Press **ALPHA** **X,θ,T** **[A]**

Press **SHIFT** **)** $[x^{-1}]$ and press **EXE**.

The GDC displays the matrix P^{-1} in fractional form.

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$$D = \begin{pmatrix} 3 & 0 \\ 0 & -2 \end{pmatrix}$$

Press **EXIT** twice

Press **F3** ▶MAT/VCT and select Mat B

Press **F3** DIM

Change the dimensions of the matrix to 2×2 and press **EXE**.

Enter the values of the elements of the matrix **D**.

Matrix B is displayed with dimensions 2x2. The elements are 3 and -2. The screen shows the matrix editor interface with 'ROW-OP', 'ROW', 'COLUMN', and 'EDIT' options.

$$A = P^{-1}DP$$

Press **EXIT** twice

Press **OPTN** **F2** MAT/VCT **F1** Mat

Press **ALPHA** **X,θ,T** **[A]**

Press **F1** Mat

Press **ALPHA** **log** **[B]**

Press **F1** Mat

Press **ALPHA** **X,θ,T** **[A]**

Press **SHIFT** **[x⁻¹]** and press **EXE**.

The screen shows the calculation of matrix A. It displays 'Mat A = Mat B * Mat A^-1' with the matrix A^-1 shown as a 2x2 matrix with elements 1/5 and -1/5. The result matrix A is shown as a 2x2 matrix with elements 1 and 2 in the first row, and 3 and 0 in the second row.

$$A^4 = PD^4P^{-1}$$

Press **F1** Mat

Press **ALPHA** **log** **[B]**

Press **^** 4 and press **EXE**.

$$D^4 = \begin{pmatrix} 81 & 0 \\ 0 & 16 \end{pmatrix}$$

$$A^4 = \begin{pmatrix} 1 & 2 \\ 1 & -3 \end{pmatrix} \begin{pmatrix} 81 & 0 \\ 0 & 16 \end{pmatrix} \begin{pmatrix} \frac{3}{5} & \frac{2}{5} \\ \frac{1}{5} & -\frac{1}{5} \end{pmatrix}$$

The screen shows the calculation of matrix A^4. It displays 'Mat B^4' with the result matrix A^4 shown as a 2x2 matrix with elements 81 and 0 in the first row, and 0 and 16 in the second row.