Chapter 9 / **Example 13**

Diagonalisation and powers of a matrix

a Find the diagonalization of $\boldsymbol{A} = \begin{pmatrix} 1 & 2 \\ 3 & 0 \end{pmatrix}$

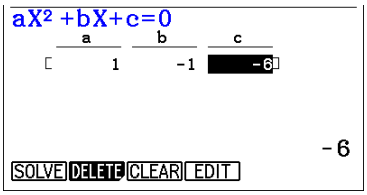
b Hence find an expression for $\boldsymbol{A}^4$ in the form $\boldsymbol{PD}^4\boldsymbol{P}^{-1}$.

c Find an expression for $\boldsymbol{A}^4$ as a product of 3 matrices with no exponents.

$|A - \lambda I| = 0 \Leftrightarrow \begin{vmatrix} 1-\lambda & 2 \\ 3 & -\lambda \end{vmatrix} = 0 \Leftrightarrow \lambda^2 - \lambda - 6 = 0$

Press MENU A and press F2 POLY.

Select F1 degree 2.



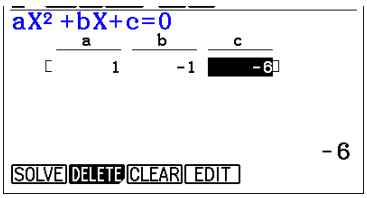
Enter the coefficients of the quadratic equation.

Press F1 SOLVE.

The calculator displays the solution $\lambda_1 = 3, \lambda_2 = -2$

The eigenvectors are

$x_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $x_2 = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$

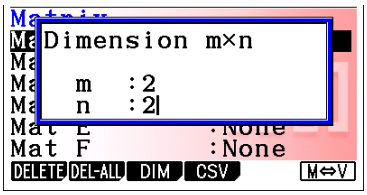


$P = \begin{pmatrix} 1 & 2 \\ 1 & -3 \end{pmatrix}$

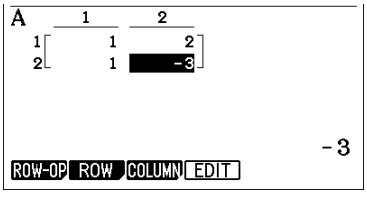
Press MENU 1 to display the Run-Matrix screen for arithmetical calculations.

Press F3 ▸MAT/VCT F3 DIM

Change the dimensions of the matrix to 2 × 2 and press EXE.



Enter the values of the elements of the matrix $\boldsymbol{P}$, using EXE to move through the matrix.



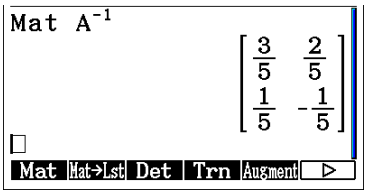
Press EXIT twice to return to the calculation screen.

Press OPTN F2 MAT/VCT F1 Mat

Press ALPHA X,θ,T [A]

Press SHIFT) [x^{-1}] and press EXE.

The GDC displays the matrix $\boldsymbol{P}^{-1}$ in fractional form.



Chapter 9 / **Example 13**

Diagonalisation and powers of a matrix

$\mathbf{D} = \begin{pmatrix} 3 & 0 \\ 0 & -2 \end{pmatrix}$

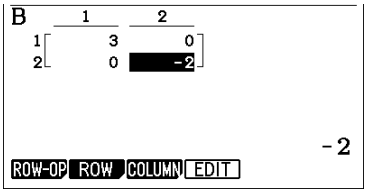
Press EXIT twice

Press F3 ▸MAT/VCT and select Mat B

Press F3 DIM

Change the dimensions of the matrix to 2 × 2 and press EXE.

Enter the values of the elements of the matrix ***D***.



$A = P^{-1}DP$

Press EXIT twice

Press OPTN F2 MAT/VCT F1 Mat

Press ALPHA X,θ,T [A]

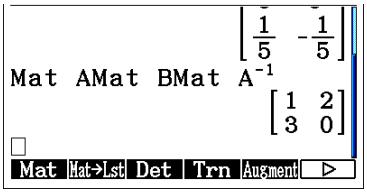
Press F1 Mat

Press ALPHA log [B]

Press F1 Mat

Press ALPHA X,θ,T [A]

Press SHIFT) [x^{-1}] and press EXE.



$A^4 = PD^4P^{-1}$

Press F1 Mat

Press ALPHA log [B]

Press ^ 4 and press EXE.

$D^4 = \begin{pmatrix} 81 & 0 \\ 0 & 16 \end{pmatrix}$

$A^4 = \begin{pmatrix} 1 & 2 \\ 1 & -3 \end{pmatrix}\begin{pmatrix} 81 & 0 \\ 0 & 16 \end{pmatrix}\begin{pmatrix} \frac{3}{5} & \frac{2}{5} \\ \frac{1}{5} & -\frac{1}{5} \end{pmatrix}$